Shinji Tsujikawa* and Bruce A. Bassett¶

* *Research Center for the Early Universe, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

¶ *Institute of Cosmology and Gravitation, University of Portsmouth, Mercantile House, Portsmouth PO1 2EG, United Kingdom*

(April 3, 2002)

We discuss the principles governing the selection of inflationary models for which preheating can affect the CMB. This is a (fairly small) subset of those models which have non-negligible entropy/isocurvature perturbations on large scales during inflation. We study new models which belong to this class - two-field inflation with negative nonminimal coupling and hybrid/double/supernatural inflation models where the tachyonic growth of entropy perturbations can lead to the variation of the curvature perturbation, $\mathcal{R}$, on super-Hubble scales. Finally we present evidence against recent claims for the variation of $\mathcal{R}$ in the absence of substantial super-Hubble entropy perturbations.

I. INTRODUCTION

The cosmological landscape is now dominated by a myriad of inflationary models [1], each with slightly different genetics but a common origin, forged in the heyday of Grand Unified field theories [2]. Inflationary models are flexible, hardy and spawn & multiply with remarkable facility - features flowing from their scalar field DNA.

And while the recent M-theory and brane-world revolution has again stimulated interest in alternatives to inflation [3] it remains true that the simplest inflationary models provide a very good fit to current cosmological observations [4]. Indeed, the Cosmic Microwave Background (CMB) and large scale structure observations presently show no particular signature - such as primordial non-Gaussianity - which might allow us to single out a particular model or falsify inflation as a paradigm. It is therefore good science to seek new ways to select the "fittest" of inflationary models.

Preheating after inflation may provide exactly such a selection rule. Preheating is probably the most violent of putative phases in cosmic history [5,6]. It requires non-perturbative quantum field theory pushed to its non-equilibrium limits [7]. The exponentially rapid population of sub- and super-Hubble wavelength modes makes it possible to produce very massive particles in large quantities and hence over-produce dangerous relics such as modulini or gravitini [8]. In addition there is some (admittedly speculative) evidence that the induced parametric growth of small-wavelength metric perturbations around the Hubble scale may lead generically to runaway production of primordial black holes (PHB) [9]. If correct this will rule out wide regions of the preheating parameter space.

At the other end of the cosmic scale, a natural question is whether preheating can affect the modes of the metric perturbations which induce the intricate anisotropies in the CMB, and if so, what are the criteria? In this paper we will discuss the conditions where preheating can lead to the growth of metric perturbations on cosmic scales.

The answer to this question appears to be intimately linked to the issue of correlations between adiabatic and entropy perturbations from multi-field inflation.

II. GENERAL PRINCIPLES

In this section we present the conditions known to be *sufficient* for preheating to affect the CMB; distilled from the recent literature [10]- [21]. Perhaps the best way to discuss the impact on the CMB is via quantities which are time-independent in the presence of only adiabatic perturbations on large scales. Such quantities are used to normalize the inflationary models to the large-angle COBE measurements of the CMB.

Two common choices for such quantities are ζ, the curvature perturbation in the constant density hypersurfaces ($\delta\rho = 0$), and the comoving curvature perturbation $\mathcal{R}$. These two coincide in the $k \to 0$ limit [22] (modulo a minus sign) and we will use $\mathcal{R}$.

The existence of entropy perturbations is crucial for preheating to affect the CMB, since in general [23]

$$\dot{\mathcal{R}} \to 3H\frac{\dot{p}}{\dot{\rho}}\mathcal{S}\,, \tag{2.1}$$

where the above limit is understood as $k \to 0$; H, p, ρ are the Hubble rate, the total pressure and energy density respectively and $\mathcal{S}$ is the total (dimensionless) entropy perturbation defined by

$$\mathcal{S} = H\left(\frac{\delta p}{\dot{p}} - \frac{\delta\rho}{\dot{\rho}}\right)\,. \tag{2.2}$$

Now in the case of two minimally coupled scalar fields φ_1 and φ_2, this can be given explicitly [22] as

$$\dot{\mathcal{R}} \to \frac{2H}{\dot{\sigma}}\dot{\theta}\delta s. \tag{2.3}$$

where σ and s are "adiabatic" and "entropy" fields, defined by

$$d\sigma = (\cos\theta) d\varphi_1 + (\sin\theta) d\varphi_2, \tag{2.4}$$

$$ds = -(\sin\theta) d\varphi_1 + (\cos\theta) d\varphi_2 \,. \tag{2.5}$$

Here θ is the angle of the trajectory in (ϕ, χ) field space, satisfying $\tan\theta = \dot\varphi_2/\dot\varphi_1$. Eq. (2.3) means that variation of $\mathcal{R}$ requires *not only* a large scale entropy perturbation δs, but also a non-straight trajectory in field space. In the minimally coupled two-field system with an effective potential $V(\varphi_1, \varphi_2)$, the Fourier modes for the adiabatic and entropy field perturbations satisfy [22]

$$\delta\ddot\sigma + 3H\delta\dot\sigma + \left(\frac{k^2}{a^2} + V_{\sigma\sigma} - \dot\theta^2\right)\delta\sigma = -2V_\sigma\Phi + 4\dot\sigma\dot\Phi + 2(\dot\theta\delta s)^{\cdot} - \frac{2V_\sigma}{\dot\sigma}\dot\theta\delta s, \tag{2.6}$$

$$\delta\ddot s + 3H\delta\dot s + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot\theta^2\right)\delta s = \frac{\dot\theta}{\dot\sigma}\frac{k^2}{2\pi G a^2}\Phi, \tag{2.7}$$

where k is a comoving momentum, a is a scale factor, G is a Newton's gravitational constant, and

$$V_{\sigma\sigma} \equiv (\cos^2\theta) V_{\varphi_1\varphi_1} + (\sin 2\theta) V_{\varphi_1\varphi_2} + (\sin^2\theta) V_{\varphi_2\varphi_2}, \tag{2.8}$$

$$V_{ss} \equiv (\sin^2\theta) V_{\varphi_1\varphi_1} - (\sin 2\theta) V_{\varphi_1\varphi_2} + (\cos^2\theta) V_{\varphi_2\varphi_2}. \tag{2.9}$$

Φ is a gravitational potential in the longitudinal gauge, satisfying

$$\dot\Phi + H\Phi = 4\pi G\dot\sigma\delta\sigma. \tag{2.10}$$

Eq. (2.10) indicates that the gravitational potential is sourced by adiabatic field perturbations.

When the effective mass of δs is light relative to the Hubble rate $H = \dot a/a$ during inflation, i.e.,

$$\mu_s^2 \equiv V_{ss} + 3\dot\theta^2 \lesssim H^2 \,, \tag{2.11}$$

the entropy field perturbation is *not* suppressed on super-Hubble scales during inflation. Then during preheating if δs is resonantly amplified due to a time-dependent effective mass, this can lead to the growth of $\mathcal{R}$ on large scales, thereby altering the power spectrum normalization or even leaving the model incompatible with the large-angle CMB. Note that the adiabatic field perturbation is sourced by the entropy field perturbation, thereby stimulating the growth of Φ through Eq. (2.10). In contrast, if the entropy perturbation is heavy during inflation, ($\mu_s^2 \gg H^2$) then $|\delta s| \sim a^{-3/2}$ and the growth during preheating means that the change of $\mathcal{R}$ is negligible before backreaction ends the resonance.

In Sec. IV we present new classes of models which have strong preheating or tachyonic growth but simultaneously have a light entropy perturbation in the preceding inflationary phase. To begin with we shall analyze the evolution of $\mathcal{R}$ in the massive chaotic inflationary scenario in the next section.

III. AN EVALUATION OF CLAIMS FOR VARYING $\mathcal{R}$ IN THE ABSENCE OF ENTROPY PERTURBATIONS

There have been recent claims by Henriques and Moorhouse (HM) [17] that $\mathcal{R}$ or ζ will vary during reheating or preheating even in the *absence of large-scale entropy perturbations* when going beyond linear perturbation theory and taking into account the quantum-to-classical transition.

If correct this would have a profound impact on inflationary cosmology [11–13]. Our aim in this section is to evaluate these claims. To do so let us consider metric preheating in the massive chaotic inflationary scenario with a standard four-leg interaction:

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 \,. \tag{3.1}$$

Strong amplification of the χ fluctuation requires a resonance parameter $q = g^2\phi^2/(4m^2) \gg 1$ at the beginning of preheating [6]. In this case the field χ is heavy during inflation relative to the Hubble rate [14], which means that χ is strongly suppressed during inflation ($\chi \sim a^{-3/2}$), thereby leading to $\dot\theta \simeq 0$ and $\langle V_{ss}\rangle \simeq \langle V_{\chi\chi}\rangle \simeq g^2\phi^2 \gg H^2$. Therefore large scale entropy perturbations are exponentially suppressed during inflation, which safeguards nonadiabatic growth of super-Hubble curvature perturbations, as found by Eq. (2.3). Hence, at the linear level, $\dot{\mathcal{R}} \simeq 0$ to high precision in this model.

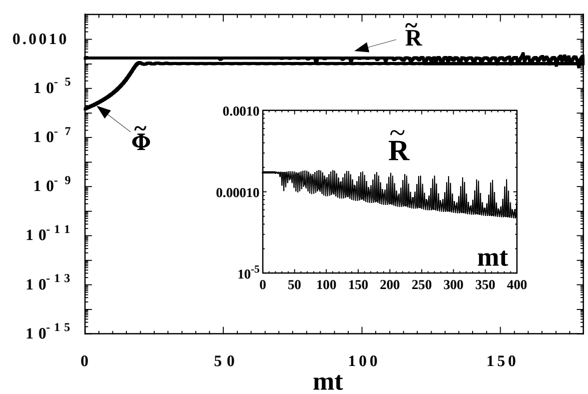

FIG. 1. The evolution of a super-Hubble curvature perturbation $\tilde{\mathcal{R}} \equiv k^{3/2}\mathcal{R}$ and the gravitational potential $\tilde\Phi \equiv k^{3/2}\Phi$ during inflation and preheating in the model (3.1) with $g = 5.0 \times 10^{-4}$. The initial conditions are chosen to be $\phi = 3M_{\rm pl}$ and $\chi = 10^{-3}M_{\rm pl}$, in which case inflation ends around $mt \sim 20$. **Inset**: The plot of $\tilde{\mathcal{R}}$ for large initial variance, $\langle\delta\phi^2\rangle = 10^{-4}\phi_0^2$.

Nevertheless we have to caution that $\dot{\mathcal{R}}$ in Eq. (2.3) is the result of the first order perturbation theory. If we include second order backreaction effects in the background evolution equations, these give rise to second order terms such as $\langle \delta\dot{\phi}^2 \rangle$ in the time derivative of $\dot{\mathcal{R}}$ [17]. Since these variances are not suppressed during inflation (due to the short-wavelength contributions), they may provide additional source terms for the curvature perturbation. This, in essence, is the origin of the claims by HM.

To check these claims we performed our numerical simulations implementing the second order backreaction effects as spatial averages in the evolution equations [6,7,19,20] and compared the evolution of the cosmological perturbations found using different (but equivalent at linear order) equations of motion.

In Fig. 1 we plot the evolution of a super-Hubble curvature perturbation using its definition, i.e., $\mathcal{R} = \Phi - H/\dot{H}(\dot{\Phi} + H\Phi)$. The variance $\langle \delta\chi^2 \rangle$ grows by parametric resonance until backreaction becomes important, while the inflaton fluctuation is not excited unless rescattering is taken into account [24].

The coherent oscillation of the inflaton condensate is destroyed once backreaction starts to dominate. This can affect the evolution of H, Φ through Eq. (2.10) and also $\mathcal{R}$. In fact the curvature perturbations exhibits small oscillations, but it remains roughly constant (see Fig. 1). When we use the first order equation (2.3), we found that $\mathcal{R}$ is conserved without oscillations.

This is in stark contrast to the simulations of HM [17] who found that $\mathcal{R}$ varies after the energy density of the inflaton condensate drops below its variance $\langle \delta\phi^2 \rangle$. According to their numerical results using similar second order approximations such as ours, the *decrease* of $\mathcal{R}$ occurs in the preheating stage.

A large difference between the two investigations is that the initial energy density of the variance $\langle \delta\phi^2 \rangle$ is smaller than ϕ_0^2 only by four orders of magnitude in Ref. [17], while we argue that the typical size is regulated to be $\langle \delta\phi^2 \rangle \sim m^2 \sim 10^{-12} M_{\rm pl}^2$ [24], which is by ten orders of magnitude smaller than ϕ_0^2.

Since $\langle \delta\phi^2 \rangle$ does not exceed ϕ_0^2 during preheating in our simulations, we do not find the decrease of $\mathcal{R}$ claimed by [17]. Using the initial conditions of HM we checked that curvature perturbations exhibit small decrease after ϕ_0^2 drops under $\langle \delta\phi^2 \rangle$ (see the inset of Fig. 1), but we still *do not* find the extensive changes reported in [17].

We did find that implementing the numerics is subtle and subject to artificial instabilities. Further, not all definitions or equations for $\mathcal{R}$ are equally suitable for numerical implementation, some being more susceptible to these instabilities. We therefore argue that $\mathcal{R}$ probably does not evolve on large scales in the absence of entropy perturbations even at second order. This is consistent with the standard view based on causality *.

*The standard view simply associates the second order terms to radiation with a temperature $T \propto \sqrt{\langle \delta\chi^2 \rangle}$. In the absence of large scale entropy perturbations in this radiation fluid no changes are induced in $\mathcal{R}$.

We end with one caveat, however: full lattice simulations show that the growth of $\delta\chi$ leads to the excitation of inflaton fluctuations via rescattering, thereby satisfying the condition $\langle \delta\phi^2 \rangle \gtrsim \phi_0^2$ around the end of preheating [24]. It would be worth investigating further whether any change in $\mathcal{R}$ occurs in such a situation, although if they do, they are likely to be small.

IV. MODELS WITH THE CMB AFFECTED BY PREHEATING

A. Chaotic inflation with self-interaction and nonminimal coupling

Achieving a light entropy perturbation during inflation is *typically* rather difficult in chaotic inflation models. However, this picture is modified if one takes into account nonminimal coupling [16]. Let us consider the quartic chaotic inflationary scenario in the presence of a nonminimally coupled scalar field χ coupled to ϕ with effective potential:

$$V = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}\xi R\chi^2 \,. \tag{4.1}$$

In this case the effective mass of χ is given by

$$m_{\rm eff}^2 \equiv g^2\phi^2 + \xi R \approx \lambda\phi^2 \left[\frac{g^2}{\lambda} + 8\pi\xi \left(\frac{\phi}{M_{pl}} \right)^2 \right] \,, \tag{4.2}$$

where we used the approximation $R \approx 12H^2$ which assumes $\chi \sim 0$ and is valid to zero order in the slow-roll parameters. Since the ξR term decreases faster than the $g^2\phi^2$ term, it is possible for χ to be light relative to H during inflation by allowing negative values of ξ. When $\xi = 0$ it was shown in Refs. [16,18–20] that super-Hubble cosmological perturbations probed by CMB experiments can be amplified around the center of the first resonance band, $g^2/\lambda = 2$ (see also Ref. [25]). For $g^2/\lambda \gtrsim 8$, using the Hartree approximation, the growth of sub-Hubble field perturbations shuts off the resonance before super-Hubble metric perturbations are enhanced [20,26].

However, this picture is modified by a negative nonminimal coupling for χ, which makes it possible to avoid the inflationary suppression of the entropy perturbation even for $g^2/\lambda \gtrsim 8$. For example, when $g^2/\lambda = 18$ and $\xi = -0.12$ shown in Fig. 2, the amplitude of the super-Hubble $\delta\chi_k$ mode at the end of inflation is larger than in the $\xi = 0$ case by about 10 orders of magnitude. In this case, large-scale curvature perturbations exhibit nonadiabatic growth after $\delta\chi$ catches up $\delta\phi$ (see Fig. 2).

When $g^2/\lambda \gg 1$, rather large negative nonminimal coupling ($\xi \lesssim -1$) is required to make the χ mass light. In this case, if the ξR term gives almost the same contribution as $g^2\phi^2$ at the beginning of inflation, it is impossible to avoid the suppression of χ due to the decrease of ξR compared to $g^2\phi^2$. When $g^2\phi^2 < -\xi R$ initially, strong